-۱

$$(a)\quad f(x) = -\frac{\pi}{۴} + \frac{h}{۲} + \sum_{k=۱}^{\infty} \left\{ \frac{۱}{\pi k^{۲}} \left[۱ + (-۱)^{k+۱} \right] \cos kx + \frac{۱}{\pi k} \left[h + (h+\pi)(-۱)^{k+۱} \right] \sin kx \right\}$$

$$(c)\quad f(x) = \sin x + \sum_{k=۱}^{\infty} \frac{۲(-۱)^{k+۱}}{k} \sin kx$$

$$(e)\quad f(x) = \frac{\sinh \pi}{\pi} \left[۱ + \sum_{k=۱}^{\infty} \frac{۲(-۱)^{k}}{۱+k^{۲}} (\cos kx - k \sin kx) \right]$$

-۲

$$(a)\quad f(x) = \sum_{k=۱}^{\infty} \frac{۲}{k} \sin kx$$

$$(b)\quad f(x) = \sum_{k=۱}^{\infty} \left(\frac{۲}{k\pi} \right) \left[۱ - ۲(-۱)^{k} + \cos \frac{k\pi}{۲} \right] \sin kx$$

$$(c)\quad f(x) = \sum_{k=۱}^{\infty} \left[۲(-۱)^{k+۱} \frac{\pi}{k} + \frac{۴}{\pi k^{۲}} \left((-۱)^{k} - ۱ \right) \right] \sin kx$$

$$(d)\quad f(x) = \sum_{k=۲}^{\infty} \frac{۲k}{\pi} \left[\frac{۱ + (-۱)^{k}}{k^{۲} - ۱} \right] \sin kx$$

-۳

$$(a)\quad f(x) = \frac{۳}{۲}\pi + \sum_{k=۱}^{\infty} \frac{۲}{\pi k^{۲}} \left[(-۱)^{k} - ۱ \right] \cos kx$$

$$(b)\quad f(x) = \frac{\pi}{۲} + \sum_{k=۱}^{\infty} \frac{۲}{\pi k^{۲}} \left[(-۱)^{k} - ۱ \right] \cos kx$$

$$(c)\quad f(x) = \frac{\pi^{۲}}{۳} + \sum_{k=۱}^{\infty} \frac{۴(-۱)^{k}}{k^{۲}} \cos kx$$

$$(d)\quad f(x) = \frac{۲}{۳\pi} + \sum_{k=۱,۲,۴,\cdots}^{\infty} \frac{۶}{\pi} \left[\frac{۱ + (-۱)^{k}}{۹ - k^{۲}} \right] \cos kx \qquad k \neq ۳$$

-۴

$$(b)\ \ f(x)=\sum_{k=۱}^{\infty}\left(\frac{۲}{k\pi}\right)\sin\frac{k\pi}{۲}\cos\left(\frac{k\pi x}{۶}\right)$$

$$(c)\ \ f(x)=\frac{۲}{\pi}+\sum_{k=۲}^{\infty}\left(\frac{۲}{k\pi}\right)\left[\frac{۱+(-۱)^k}{۱-k^۲}\right]\cos\left(\frac{k\pi x}{l}\right)$$

$$(f)\ \ f(x)=\sum_{k=۱}^{\infty}\frac{k\pi}{۱+k^۲\pi^۲}(-۱)^{k+۱}\left(e-e^{-۱}\right)\sin(k\pi x)$$

-۵

$$(a)\ \ f(x)=\sum_{k=-\infty}^{\infty}\frac{۱}{\pi}\left(\frac{۲+ik}{۴+k^۲}\right)(-۱)^k\sinh ۲\pi e^{ikx}$$

$$(b)\ \ f(x)=\sum_{k=-\infty}^{\infty}\frac{(-۱)^k}{\pi\left(۱+k^۲\right)}(-۱)^k\sinh\pi e^{ikx}$$

$$(d)\ \ f(x)=\sum_{k=-\infty}^{\infty}(-۱)^k\left(\frac{i}{k\pi}\right)e^{ik\pi x}$$

-۶

$$(a)\ \ f(x)=\frac{\pi}{۸}\pi+\sum_{k=۱}^{\infty}\left[\frac{۱}{۲\pi k^۲}\left\{(-۱)^k-۱\right\}\cos kx+\frac{(-۱)^{k+۱}}{۲k}\sin kx\right]$$

-۷

$$(a)\ \ f(x)=\frac{l^۲}{۳}+\sum_{k=۱}^{\infty}۴(-۱)^k\left(\frac{۱}{k\pi}\right)\cos\left(\frac{k\pi x}{l}\right)$$

-۸

$$(a)\ \ \sin^۲ x=\sum_{k=۱,۳,۴,\cdots}^{\infty}\frac{۴(۱-\cos k\pi}{k\pi\left(۴-k^۲\right)}\sin kx$$

$$(b)\ \ \cos^۲ x=\sum_{k=۱,۳,۴,\cdots}^{\infty}\frac{۲}{k\pi}\left(\frac{۱-k^۲}{۴-k^۲}\right)(۱-\cos k\pi)\sin kx$$

$$(b)\ \ \sin x\cos x=\sum_{k=۱,۳,۴,\cdots}^{\infty}\frac{۲}{\pi}\left(\frac{۱-\cos k\pi}{۴-k^۲}\right)\cos kx$$

۹-

$$(a)\quad \frac{x^{۲}}{۴} = \frac{\pi^{۲}}{۱۲} - \sum_{k=۱}^{\infty} \frac{(-۱)^{k+۱}}{k^{۲}} \cos kx$$

$$(c)\quad \int_{۰}^{\infty} \ln\left(۲\cos\frac{x}{۲}\right) x = \sum_{k=۱}^{\infty} (-۱)^{k+۱} \frac{\sin kx}{k^{۲}}$$

$$(e)\quad \frac{\pi}{۲} - \frac{۴}{\pi} \sum_{k=۱}^{\infty} \frac{\cos(۲k-۱)x}{(۲k-۱)^{۲}} = \begin{cases} -x & -\pi < x < ۰ \\ x & ۰ < x < \pi \end{cases}$$

$$(a)\quad f(x,y)=\frac{۱۶}{\pi^{۲}}\sum_{m=۱,۳,\cdots}^{\infty}\sum_{n=۱,۳,\cdots}^{\infty}\left(\frac{۱}{mn}\right)\sin mx\sin ny$$

$$(c)\quad f(x,y)=\frac{\pi^{۴}}{۹}+\frac{۱}{۲}\sum_{m=۱}^{\infty}\frac{۸}{۳}\pi^{۲}\frac{(-۱)^{m}}{m^{۲}}\cos mx+\frac{۱}{۲}\sum_{n=۱}^{\infty}\frac{۸}{۳}\pi^{۲}\frac{(-۱)^{n}}{n^{۲}}\cos ny$$
$$+\sum_{m=۱}^{\infty}\sum_{n=۱}^{\infty}\frac{۱۶(-۱)^{m+n}}{m^{۲}n^{۲}}\cos mx\cos ny$$

$$(e)\quad f(x,y)=\sum_{m=۱}^{\infty}\frac{۲(-۱)^{m+۱}}{m}\sin mx\sin y$$

$$(g)\quad f(x,y)=\sum_{m=۱}^{\infty}\sum_{n=۱}^{\infty}d_{mn}\sin\left(\frac{m\pi x}{۱}\right)\sin\left(\frac{n\pi y}{۲}\right)$$

که

$$\begin{aligned}
d_{mn} &= \tfrac{۴}{۱/۲}\int_{۰}^{۲}\int_{۰}^{۱} xy\sin(m\pi x)\sin\left(\tfrac{n\pi y}{۲}\right)xy\\
&= ۲\int_{۰}^{۲}\left[\tfrac{\sin m\pi x}{m^{۲}\pi^{۲}}-\tfrac{x\cos m\pi x}{m\pi}\right]_{۰}^{۱} y\sin\left(\tfrac{n\pi y}{۲}\right)y\\
&= \tfrac{-۲(-۱)^{m}}{m\pi}\int_{۰}^{۲} y\sin\left(\tfrac{n\pi y}{۲}\right)y=\tfrac{-۲(-۱)^{m}}{m\pi}\left(\tfrac{-۴(-۱)^{n}}{n\pi}\right)\\
&= \tfrac{۸(-۱)^{m+n}}{\pi^{۲}mn}
\end{aligned}$$

$$(h)\quad f(x,y)=\left(\frac{۱۶}{\pi^{۲}}\right)\sum_{m=۱}^{\infty}\sum_{n=۱}^{\infty}[(۲m-۱)(۲n-۱)]^{-۱}\sin\left[\frac{(۲m-۱)\pi x}{a}\right]$$
$$\times\sin\left[\frac{(۲n-۱)\pi y}{a}\right]$$

$$(i)\quad f(x,y)=\left(\frac{\sin ۲}{\pi}\right)\sum_{m=۱}^{\infty}\frac{(-۱)^{m+۱}}{m}\sin m\pi x$$
$$+\left(\frac{۸\sin ۲}{\pi}\right)\sum_{m=۱}^{\infty}\sum_{n=۱}^{\infty}\left[\frac{(-۱)^{m+n+۱}}{m\left(۴-\pi^{۲}n^{۲}\right)}\right]\sin(m\pi x)\cos\left(\frac{n\pi y}{۲}\right)$$

$$(j)\quad f(x,y)=\frac{۲}{۳}\pi^{۲}\sum_{m=۱}^{\infty}\frac{(-۱)^{m+۱}}{m}\sin mx+\sum_{m=۱}^{\infty}\sum_{n=۱}^{\infty}\frac{۸(-۱)^{m+n+۱}}{mn^{۲}}\sin mx\sin ny$$

$$(k)\quad f(x,y)=\frac{\pi^{۴}}{۹}+\left(\frac{۴\pi^{۲}}{۳}\right)\left[\sum_{m=۱}^{\infty}\frac{(-۱)^{m}}{m^{۲}}\cos mx+\sum_{n=۱}^{\infty}\frac{(-۱)^{n}}{n^{۲}}\cos ny\right]$$
$$+۱۶\sum_{m=۱}^{\infty}\sum_{n=۱}^{\infty}\frac{(-۱)^{m+n}}{m^{۲}n^{۲}}\cos mx\cos ny$$

-۲۰

$$(a)\quad b_{۲n} = ۰,\quad b_{۲n+۱} = \frac{۸}{\pi(۲n+۱)^۳}$$

$$f(x) = x(\pi - x) = \frac{۸}{\pi}\left(\frac{\sin x}{۱^۳} + \frac{\sin ۳x}{۳^۳} + \frac{\sin ۵x}{۵^۳} + \cdots\right)$$

(b) جهت یافتن مجموع سری ها قرار دهید $x = \frac{\pi}{۴}$ و $x = \frac{\pi}{۲}$

-۲۱

$$(a)\quad b_n = \frac{۲}{\pi}\int_۰^\pi f(x)\sin nx x = \left(\frac{۸}{n^۲\pi^۲}\right)\sin\left(\frac{n\pi}{۲}\right),\qquad n = ۰, ۱, ۲, \cdots$$

$$b_{۲n} = ۰,\quad b_n = \frac{۸(-۱)^n}{\pi^۲(۲n+۱)^۲}$$

(b) قرار دهید $x = \frac{\pi}{۲}$

-۲۲

$$(a)\quad f(x)=\sum_{n=۱}^{\infty} b_n \sin\left(\frac{n\pi x}{a}\right),$$

$$b_n=\frac{۲}{a}\int_{۰}^{a}\sin\left(\frac{n\pi x}{a}\right)x=\frac{۲}{n\pi}(۱-\cos n\pi)$$

$$=\frac{۲}{n\pi}\left[۱-(-۱)^n\right]=\begin{cases}\frac{۴}{n\pi} & \text{زوج } n\\ ۰ & \text{فرد } n\end{cases}$$

$$f(x)\sim\frac{۴}{\pi}\left[\sin\left(\frac{\pi x}{a}\right)+\frac{۱}{۳}\sin\left(\frac{۳\pi x}{a}\right)+\frac{۱}{۵}\sin\left(\frac{۵\pi x}{a}\right)+\cdots\right]$$

$$f(x)\sim\frac{۱}{۲}a_{۰}+\sum_{n=۱}^{\infty}a_n\cos\left(\frac{n\pi x}{a}\right),\qquad a_{۰}=۲$$

$$a_n=\frac{۲}{n\pi}(\sin n\pi-۰)=۰,\qquad n\neq ۰$$

$$۱=۱+۰\cdot\cos\left(\frac{n\pi}{a}\right)+۰\cdot\cos\left(\frac{۲n\pi}{a}\right)+\cdots$$

$$(b)\quad f(x)=\sum_{n=۱}^{\infty}(-۱)^{n+۱}\frac{۲a}{n\pi}\sin\left(\frac{n\pi x}{a}\right)$$

$$f(x)\sim\frac{۱}{۲}a_{۰}+\sum_{n=۱}^{\infty}a_n\cos\left(\frac{n\pi x}{a}\right)\qquad a_{۰}=\frac{۲}{a}\int_{۰}^{a}xx=a$$

$$a_n=\frac{۲}{a}\int_{۰}^{a}x\cos\left(\frac{n\pi x}{a}\right)x=\frac{۲a}{n^{۲}\pi^{۲}}\left((-۱)^n-۱\right)$$

$$=\begin{cases}۰ & \text{زوج } n\\ -\frac{۴a}{n^{۲}\pi^{۲}} & \text{فرد } n\end{cases}$$

-۲۳

$$(a)\quad a_. = \frac{۱}{a}\int_{-a}^{a} xx = ۰$$

$$a_n = \frac{۱}{a}\int_{-a}^{a} x\cos\left(\frac{n\pi x}{a}\right) x = \left[\frac{x}{n\pi}\sin\left(\frac{n\pi x}{a}\right) + \frac{a}{n^{۲}\pi^{۲}}\cos\left(\frac{n\pi x}{a}\right)\right]_{-a}^{a}$$

$$= \frac{a}{n^{۲}\pi^{۲}}\left(\cos n\pi - \cos(-n\pi)\right) = ۰$$

$$b_n = \frac{۱}{a}\int_{-a}^{a} x\sin\left(\frac{n\pi x}{a}\right. x$$

$$= \left[\frac{-x}{n\pi}\cos\left(\frac{n\pi x}{a}\right) + \frac{a}{n^{۲}\pi^{۲}}\sin\left(\frac{n\pi x}{a}\right)\right]_{-a}^{a}$$

$$= \frac{-a}{n\pi}\cos n\pi + \frac{-a}{n\pi}\cos(-n\pi) = (-۱)^{n+۱}\left(\frac{۲a}{n\pi}\right)$$

$$(c)\quad a_. = \frac{۱}{۲\pi}\int_{۰}^{۲۱} f(x)x = \frac{۱}{۲\pi}\int_{\pi}^{۲۱} x = \frac{۱}{۲}$$

$$a_n = \frac{۱}{\pi}\int_{۰}^{۲۱} f(x)\cos nxx = \frac{۱}{\pi}\int_{\pi}^{۲۱}\cos nxx = ۰,\quad n \text{ برای هر}$$

$$b_n = \frac{۱}{\pi}\int_{۰}^{۲۱} f(x)\sin nxx = \frac{۱}{\pi}\int_{\pi}^{۲۱}\sin nxx$$

$$= \frac{۱}{n\pi}\left[-۱ + (-۱)^n\right] = \begin{cases} ۰ & n \text{ زوج} \\ -\frac{۲}{n\pi} & n \text{ فرد} \end{cases}$$

$$f(x) = \frac{۱}{۲} - \frac{۲}{\pi}\sum_{k=۰}^{\infty}\frac{\sin(۲k+۱)x}{۲k+۱}$$