

فصل ۱

مروری بر مطالب مورد نیاز

۱.۱ مقدمه

در این فصل ابتدا به بیان مفاهیم پایه‌ای مورد نیاز، شامل مباحثی از جبر خطی، آنالیز محدب و بهینه‌سازی می‌پردازیم. سپس به طور خلاصه دو الگوریتم کارای موجود برای حل زیر مساله ناحیه اعتماد در ابعاد بزرگ را شرح می‌دهیم.

۲.۱ مقدماتی از جبر خطی

۱.۲.۱ مقادیر ویژه و بردارهای ویژه

قضیه زیر کوچکترین و بزرگترین مقدار ویژه ماتریس متقارن A را به صورت یک مساله بهینه‌سازی بیان می‌کند. قضیه ۱.۲.۱ ([۱]).

$$\lambda_{\min}(A) = \min_{s.t. \quad ||x||^2 = 1} x^T A x$$

$$\lambda_{\max}(A) = \max_{s.t. \quad ||x||^2 = 1} x^T A x$$

مجموعه‌های زیر مخروط‌های محدب هستند
[۸، ۹، ۱۰، ۱۱، ۱۲، ۱۳، ۱۴، ۱۵، ۱۶، ۱۷، ۱۸، ۱۹، ۲۰] که پایه و اساس همه آن‌ها را می‌توان در شرایط بهینگی لازم و کافی آن دانست. در این بین الگوریتم کلاسیک مور و سورنسن

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