

$$\begin{aligned}
\Delta_{\mathfrak{r}}(t) &= F_{H_{\mathfrak{r}}}(t) \int_0^t \int_0^v F_{Y_{\mathfrak{r}}}(t - \delta(u)) F_{Y_{\mathfrak{r}}}(t - \delta(v)) \eta(u, v) dudv \quad (1) \\
&\quad + F_{H_{\mathfrak{r}}}(t) \int_0^t \int_v^t F_{Y_{\mathfrak{r}}}(t - \delta(u)) F_{Y_{\mathfrak{r}}}(t - \delta(v)) \eta(u, v) dudv \\
&= F_{H_{\mathfrak{r}}}(t) \int_0^t \int_0^v F_{Y_{\mathfrak{r}}}(t - \delta(u)) F_{Y_{\mathfrak{r}}}(t - \delta(v)) \eta(u, v) dudv \\
&\quad + F_{H_{\mathfrak{r}}}(t) \int_0^t \int_0^u F_{Y_{\mathfrak{r}}}(t - \delta(u)) F_{Y_{\mathfrak{r}}}(t - \delta(v)) \eta(u, v) dudv \\
&= F_{H_{\mathfrak{r}}}(t) \int_0^t \int_0^v F_{Y_{\mathfrak{r}}}(t - \delta(u)) F_{Y_{\mathfrak{r}}}(t - \delta(v)) \eta(u, v) dudv \\
&\quad + F_{H_{\mathfrak{r}}}(t) \int_0^t \int_0^v F_{Y_{\mathfrak{r}}}(t - \delta(v)) F_{Y_{\mathfrak{r}}}(t - \delta(u)) \eta(v, u) dudv \\
&= F_{H_{\mathfrak{r}}}(t) \int_0^t \int_0^v \xi_{\mathfrak{r}}(u, v) \eta(u, v) dudv
\end{aligned}$$