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RENORMALIZATION GROUP EFFECTS IN THE CONFORMAL SECTOR OF 4D QUANTUM GRAVITY WITH MATTER

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Abstract

We discuss the "gravitationally dressed" beta functions in the Gross–Neveu model interacting with 2d Liouville theory and in $SU(N)$ gauge theory interacting with the conformal sector of 4d quantum gravity. Among the effects that we suggest may feel the gravitational dressing are the minimum of the effective potential and the running of the gauge coupling.

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The influence of quantum gravitational effects on the renormalization group (RG) is expected to be very complicated. Already in the case of a quantum field theory in curved spacetime, the RG is getting quite nontrivial [1]. For example, many new couplings appear which are absent in flat space.

Recently it has been discovered [2] that 2d quantum gravity gives a "gravitational dressing" of the matter beta functions. Then, the RG equations of matter are modified even in the situation when the divergences are formally the same as in the absence of quantum gravity [2,3,4].

In the present note we try to find a 4d analog of the "gravitational dressing" of the matter beta function using the effective theory of the conformal factor [5] as an example. We shall begin by giving an alternative, RG based, derivation of this theory. We then consider the RG dressing of the beta function in the case of the 2d Gross–Neveu model coupled to gravity. Finally we apply the same method to 4d $SU(N)$ gauge theory coupled to the conformal sector of Quantum Gravity (QG).

We begin by considering a massive self-interacting scalar theory in curved spacetime. Our first purpose is to show that the conformal sector of quantum gravity can be fixed by the requirement of multiplicative renormalizability of such a theory, with the metric treated as an external field. Let us therefore start from the action

$$S(\varphi) = -\int d^4x \, \sqrt{g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{1}{2}(m^2 + \xi R)\varphi^2 + \frac{f}{4!}\varphi^4 \right] , \tag{1}$$

In what follows we will restrict our attention to metrics of the form

$$g_{\mu\nu} = e^{2\sigma} \eta_{\mu\nu} , \tag{2}$$

where $\eta_{\mu\nu}$ is the flat metric and σ is the conformal factor, which will be the only dynamical component of the gravitational field. Defining

$$\phi = e^{\sigma} \varphi , \tag{3}$$

the action (1) can be written as

$$S(\sigma, \rho) = -\int d^4x \, \left[\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} m^2 e^{2\sigma} \phi^2 + \frac{f}{4!}\phi^4 + \frac{1}{2}\tau \phi^2 (\partial^2 \sigma + \eta^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma) \right] , \tag{4}$$

where $\tau = 1 - 6\xi$. Note some nonstandard terms describing the interaction of the scalar field ϕ with σ. All these terms disappear for $m^2 = 0$, $\tau = 0$, *i.e.* when (1) is invariant under Weyl transformations of $g_{\mu\nu}$.

We are going to study the renormalization of the theory (4) regarded as a quantum theory in flat space coupled to a scalar background field σ. According to the standard renormalization formalism in a background field (see for example [1]) in order to guarantee the multiplicative renormalizability of the theory one has to add to the action (4) the action of the external fields. Considerations of dimensionality, covariance and explicit loop calculations fix the external field action to be of the form

$$S_{\text{ext}} = \int d^4x \left[-\frac{Q^2}{(4\pi)^2} (\partial^2 \sigma)^2 - \zeta \left[2(\partial_\mu \sigma)^2 \partial^2 \sigma + (\partial \sigma)^4 \right] + \gamma e^{2\sigma} (\partial_\mu \sigma)^2 - \lambda e^{4\sigma} \right] . \tag{5}$$

This is exactly the same action that one gets as a result of integrating over the conformal anomaly [7,5]. (For a recent very interesting discussion of the general structure of the conformal anomaly in d dimensions, see [8]). In [5] the coupling constants of (5) were fixed in terms of the coefficients of the conformal anomaly, and $S_{\rm ext}$ was used to describe the IR region of QG. We have a completely different interpretation of $S_{\rm eff}$ – its form is required by the condition of multiplicative renormalizability. The effective potential for the conformal factor in the theory (4) was discussed in [6].

Let us discuss now the renormalization of the theory under discussion. By the standard calculation of one loop divergences, one can find the beta functions, and from there the effective running coupling constants

$$\begin{array}{ll} f(t) = \frac{f}{A(t)} & m^2(t) = m^2 A(t)^{-1/3} \\ \tau(t) = \tau A(t)^{-1/3} & \\ \zeta(t) = \zeta + \frac{\tau^2}{2f} B(t) & Q^2(t) = Q^2 + \frac{(4\pi)^2}{2f} \tau^2 B(t) \\ \gamma(t) = \gamma + \frac{m^2 \tau}{f} B(t) & \lambda(t) = \lambda + \frac{m^4}{2f} B(t) \end{array}$$

where $A(t) = 1 - \frac{3f}{(4\pi)^2} t$ and $B(t) = A(t)^{1/3} - 1$. One sees that both in the IR limit $t \to -\infty$, where the theory is asymptotically free, and in the UV limit, where the problem of zero charge appears, the effective couplings tend to their conformally invariant values. At the same time the couplings of $S_{\rm ext}$ grow infinitely with $|t|$.

We are interested in the physical scaling of 4d IR quantum gravity with matter, *i.e.* the theory with action (5) where σ is also a quantum field now.

Before doing this, we discuss first the analogous situation in $d = 2$, namely the case of Liouville theory. The effective Lagrangian for the Liouville part of the induced quantum gravity may be written

$$L = -\frac{Q^2}{4\pi} (\nabla \sigma)^2 - 2\lambda e^{2\sigma} \ , \tag{6}$$

where the conformal gauge $g_{\mu\nu} = e^{2\sigma} \eta_{\mu\nu}$ has been used, $Q^2 = \frac{1}{12}(25 - c)$ and c is the central charge. Quantizing σ one finds that there is an anomalous scaling behaviour. It can be defined by saying that the conformal factor e^σ acquires a scaling dimension α (*i.e.* putting $\hat{\sigma} = \alpha\sigma$), calculating the exact beta function for λ and demanding the vanishing of this beta function for $\lambda \neq 0$. This gives [9]

$$\alpha = \frac{1 - \sqrt{1 - \frac{2}{Q^2}}}{\frac{1}{Q^2}} \ , \tag{7}$$

where the negative branch of the square root is to be chosen. The classical scaling $\alpha = 1$ is obtained in the limit $Q^2 \to \infty$ (no quantum gravity). Substituting Q^2 in (7) one has

$$\alpha = Q \left(Q - \sqrt{Q^2 - 2} \right) = \sqrt{\frac{25 - c}{12}} \left(\sqrt{\frac{25 - c}{12}} - \sqrt{\frac{1 - c}{12}} \right) \tag{8}$$

showing that the critical exponent is real only for $Q^2 > 2$. In terms of the critical exponent, the renormalized dimensionless cosmological constant satisfies the exact renormalization group equation [3,10]

$$\frac{\partial \lambda}{\partial \log \mu} = -2\alpha\lambda \tag{9}$$

with the solution

$$\lambda(\mu) \sim \mu^{-2\alpha} \ . \tag{10}$$

If $c \to -\infty$ (which corresponds to the absence of quantum gravity), $\lambda(\mu) \to \mu^{-2}$.

Now let us add to the 2d induced quantum gravity a matter action corresponding for example to the Gross–Neveu model [11] (The interaction of this model with dilaton gravity has been considered recently in [12]). The Lagrangian is given by

$$L = \bar{\psi}(i\partial)\psi - \frac{1}{2}\sigma^2 - g\sigma\bar{\psi}\psi \ , \tag{11}$$

where σ is a scalar and ψ is an N component massless fermion. In the leading order of the $1/N$ approximation, the beta function of this theory does not change in the presence of gravity and is given by

$$\beta_g = -\frac{\lambda g}{2\pi}, \qquad \lambda = Ng^2 \ . \tag{12}$$

The running coupling constant has the asymptotically free form $g^2(t) = \frac{g^2}{\left(1+\frac{\lambda t}{\pi}\right)}$. The gravitational dressing of the beta function (12), can be derived by the following argument [3]. In the presence of quantum gravity the physically meaningful beta functions are not the derivatives of the coupling constants with respect to the mass scale μ, but rather the derivatives with respect to some other scale coming from the gravitational sector [10]. In [3] the cosmological term was chosen, but the result is universal. Then

$$\beta_g^G = \frac{\partial g}{\partial \log \lambda^{-1/2}} = \frac{\partial g}{\partial \log \mu}\frac{\partial \log \mu}{\partial \log \lambda^{-1/2}} = \frac{1}{\alpha}\beta_g \ . \tag{13}$$

This is the "gravitational dressing" of the beta function [2,3,4]. It is interesting to understand how these considerations explicitly modify the running coupling constant and effective potential. From (13) we get

$$\frac{dg(t)}{dt} = -\frac{\lambda(t)g(t)}{2\pi\alpha} \tag{14}$$

and the "gravitationally dressed" running coupling is $g^2(t) = \frac{g^2}{\left(1+\frac{\lambda t}{\pi\alpha}\right)}$. Hence the value of the pole in the IR region changes from $t = -\frac{\pi}{\lambda}$ (no QG) to $t = -\frac{\pi\alpha}{\lambda}$ (with QG). Now let us consider the one loop effective potential in this theory. It may be found as a solution of the RG equation [13]

$$\left[\mu\frac{\partial}{\partial\mu} + \tilde{\beta}^G\frac{\partial}{\partial g} - \tilde{\gamma}^G\sigma\frac{\partial}{\partial\sigma}\right]V = 0 \ . \tag{15}$$

Here

$$\tilde{\beta}^G = \frac{1}{1-\gamma^G}\beta^G \ , \qquad \tilde{\gamma}^G = \frac{1}{1-\gamma^G}\gamma^G \ , \tag{16}$$

as usual. Note that in these equations we have to use the gravitationally dressed RG functions. We can solve the RG equation (15) and find

$$V = \frac{1}{2}\sigma^2 + \frac{\lambda}{4\pi\alpha}\sigma^2\left[\log\frac{\sigma^2}{\mu^2} - 3\right] \ . \tag{17}$$

The effect of the gravitational dressing is the factor $1/\alpha$ in the second term. In principle, it could lead to some observable physical effect. In fact, the minimum of the potential and the fermionic mass are given by

$$\sigma_m = \mu \exp\left(1 - \frac{\pi\alpha}{\lambda}\right) \ , \qquad M_f = g\sigma_m \ . \tag{18}$$

Both these values feel the effect of gravity. Of course these considerations are somehow formal (just as in [2,3,4]) because in this model $c > 1$ (for $N > 1$). However, in principle, one could add to the system some exotic free conformal field theory to lower the value of c without changing the dynamics. In any case, this model gives some indication of what one can expect to find in more realistic situations, like the d=4 case that we now return to.

We work within the framework of the effective dynamics for the conformal sector which was discussed first in [5], as a 4d analog of Liouville theory and was rederived from a different viewpoint in the beginning of this paper. We consider the effective action (5) for the conformal factor in the IR stable fixed point $\zeta = 0$, which presumably corresponds to the IR sector of QG. The Lagrangian is therefore

$$L = -\frac{Q^2}{(4\pi)^2}(\partial^2\sigma)^2 + \gamma e^{2\sigma}(\partial_\mu\sigma)^2 - \lambda e^{4\sigma} \ . \tag{19}$$

where $Q^2 = -32\pi^2 b'$, and b' is the coefficient of the Gauss-Bonnet term in the conformal anomaly. Proceeding from here as in the two dimensional case one can put $\sigma = \alpha\hat{\sigma}$, find the exact beta functions for γ and λ and from the condition of vanishing beta functions derive the anomalous scaling dimension of e^σ [5]:

$$\alpha = \frac{1 - \sqrt{1 - \frac{4}{Q^2}}}{\frac{2}{Q^2}} \ , \tag{20}$$

This critical exponent is real for $Q^2 > 4$, which is the standard situation in $d = 4$.

In terms of this critical exponent we have again an exact RG equation for the renormalized cosmological constant

$$\frac{\partial\lambda}{\partial\log\mu} = -4\alpha\lambda \tag{21}$$

giving $\lambda \sim \mu^{-4\alpha}$. In the limit of no QG, $Q^2 \to \infty$, $\alpha \to 1$.

By analogy with the 2d case we define the physical mass scale as $\lambda^{-1/4}$ (the same results would be obtained if we used instead $\gamma^{-1/2}$). Let us now couple the theory with Lagrangian (5) to some massless matter theory consisting, for simplicity, of spinors and $SU(N)$ gauge fields. We avoid scalar and Yukawa couplings in this context, since their beta functions are known to receive contributions from the spin 2 sector of quantum gravity (for an example in the context of R^2–gravity see [15,1]). On the contrary, the beta function of the gauge coupling does not change in the presence of gravity [14,15]. Hence, we expect that transverse spin 2 degrees of freedom may be safely neglected in this case.

Writing the Lagrangian of spinor and gauge fields in an external metric of the form (2) and making the appropriate rescalings of the fields as in (3), one can see that this theory does not couple to the scalar σ. Hence the one-loop beta function for the gauge coupling is the same as in flat space:

$$\beta_{g^2} = \frac{\partial g^2}{\partial \log \mu} = -a^2 g^4 \ , \tag{22}$$

where the constant a^2 depends on the details of the theory.

As in 2d the effect of QG will be to replace the beta function (22) by the gravitationally dressed beta function

$$\beta^G_{g^2} = \frac{\partial g^2}{\partial \lambda^{-1/4}} = \frac{\partial g^2}{\partial \log \mu} \frac{\partial \log \mu}{\partial \log \lambda^{-1/4}} = \frac{1}{\alpha} \beta_{g^2} \ . \tag{23}$$

The gravitationally dressed running gauge coupling is

$$g(t)^2 = \frac{g^2}{\left(1 + \frac{a^2 g^2 t}{\alpha}\right)} \ . \tag{24}$$

For an $SU(N)$ model, taking into account also the contributions to Q^2 coming from Einstein gravity or Weyl gravity, and from the σ sector, we have

$$Q^2 = \frac{1}{180} \left(11 N_F + 62 N_V + n\right) \ , \qquad a^2 = \frac{1}{(4\pi)^2} \left[\frac{22}{3} N - \frac{4}{3} \sum_{\rm F} T(R) \right] \ , \tag{25}$$

where the sum in the last equation is over all fermion representations and $n = 1383$ for Einstein gravity, $n = 1583$ for Weyl gravity [16]. For example in the case of the standard model one has $\alpha = 1.13$ and 1.11 respectively. We are neglecting here the scalar contribution to Q^2, which is very small with respect to the rest. As we see, the gravitational dressing of the gauge coupling beta function has quite modest effects, of the order of few percents. Taking this effect into account in the running of the gauge coupling constants α_1, α_2, α_3 corresponding to the groups $U(1)$, $SU(2)$ and $SU(3)$, and imposing that they meet at some unification scale, we find an increase in this scale of a factor of the order 2, compared to the case in which gravity is neglected. This is in principle an observable effect.

We would like to conclude with a remark on the effective potential in the $SU(N)$ gauge theory. Consider the composite field $K = A_\mu A^\mu$. The effective potential for K has been calculated in [17]:

$$V = -\frac{3n}{(4\pi)^2\alpha} K^2 \log \frac{K}{\mu^2} \ , \tag{26}$$

in the Landau gauge (n is the dimension of the group). Due to the gravitational dressing, this potential is modified by the appearance of a factor $1/\alpha$. As in the case of the Gross–Neveu model, this may lead to a modification of the dynamically generated mass, which was discussed in [17] in the case $\alpha = 1$.

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